

Leptogenesis within a generalized quark-lepton symmetry

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Quark-lepton symmetry has been shown to be inconsistent with baryogenesis via leptogenesis in natural schemes of the see-saw mechanism. Within the phenomenological approach of textures, we relax this strict symmetry and propose weaker conditions, namely models of the neutrino Dirac mass matrix M_D which have the same hierarchy as the matrix elements of M_u . We call this guide-line generalized *hierarchical* quark-lepton symmetry. We consider in detail particular cases in which the moduli of the matrix elements of M_D are equal to those of M_u . Within the phenomenological approach of textures, we try for the heavy Majorana mass matrix diagonal and off-diagonal forms. We find that an ansatz for M_D preserving the hierarchy, together with an off-diagonal model for the heavy Majorana neutrino mass, is consistent with neutrino masses, neutrino mixing and baryogenesis via leptogenesis for an intermediate mass scale $m_R \sim 10^{12}$ GeV. The preservation of the hierarchical structure could come from a possible symmetry scheme.

I. INTRODUCTION

The seesaw mechanism [1] can explain the smallness of neutrino masses and is consistent with large lepton mixing [2]. In a nutshell, it is based on the existence of very heavy right-handed Majorana neutrinos. A cosmological consequence is the generation of a baryon asymmetry in the universe by means of the out-of-equilibrium decays of the heavy right-handed neutrinos to leptons and $SU(2)_L$ Higgs bosons, which create a lepton asymmetry, partially converted to a baryon asymmetry by electroweak sphalerons, a mechanism known as baryogenesis via leptogenesis [3].

Recently, much progress has been done in the study of leptogenesis, especially in the exploration of flavor effects [4]. In the present paper we turn again to the link between leptogenesis and fermion mass matrices. We are interested in the compatibility of quark-lepton symmetry with leptogenesis. In previous papers this compatibility has been strongly questioned [5]. In Ref. [6] it is achieved only in the case of some degeneracy of the right-handed neutrino masses. This was also argued in Ref. [7]. Often an inverse seesaw formula was used. In order to further explore this subject, we adopt here a typical form of the quark mass matrices [8], together with minimal models for the Majorana mass matrix [9], and the direct seesaw formula. In a second step we relax quark-lepton symmetry, and adopt a weaker hypothesis, namely keeping the hierarchy of the Dirac mass matrix elements, with the moduli of the elements of M_D being equal to the moduli of the elements of M_u .

II. MASS MATRICES

According to the seesaw mechanism, the effective mass matrix of neutrinos is given by the formula

$$M_\nu \simeq M_D M_R^{-1} M_D, \quad (1)$$

where M_D is the Dirac mass matrix and M_R the Majorana mass matrix. For $M_R \gg M_D$, we have $M_\nu \ll M_D$.

Our starting point for fermion mass matrices is the following symmetric forms of quark mass matrices [8, 10], for which we simply give an order of magnitude of the matrix elements,

$$M_u \simeq \begin{pmatrix} 0 & i\epsilon_u^3 & \epsilon_u^4 \\ i\epsilon_u^3 & \epsilon_u^2 & \epsilon_u^2 \\ \epsilon_u^4 & \epsilon_u^2 & 1 \end{pmatrix} m_t, \quad (2)$$

$$M_d \simeq \begin{pmatrix} 0 & \epsilon_d^3 & \epsilon_d^4 \\ \epsilon_d^3 & \epsilon_d^2 & \epsilon_d^2 \\ \epsilon_d^4 & \epsilon_d^2 & 1 \end{pmatrix} m_b, \quad (3)$$

with $\epsilon_u^2 = m_c/m_t$, $\epsilon_d^2 = m_s/m_b$, i.e. $\epsilon_u \simeq 0.05$ and $\epsilon_d \simeq 0.15$, which agree with the mass spectrum of the quarks and the CKM mixing matrix. Then, simple quark-lepton symmetry leads to the relations

$$M_D = M_u, \quad M_e = M_d \quad (4)$$

for example in $SO(10)$ with Higgses transforming as **10** representations. Indeed, $SO(10)$ is a favored scenario for neutrino mass and leptogenesis, since one has right-handed neutrinos with heavy Majorana masses and $B - L$ spontaneous symmetry breaking. As it is well-known, the relation $M_e = M_d$ can be naturally modified in $SO(10)$ with a **126** representation in order to have -3 Clebsch-Gordan coefficient, coming from color, in the (2,2) entry for M_e relatively to M_d , that yields the better relation $m_s = m_\mu/3$ at the unification scale [11].

For the right-handed neutrino we take minimal mass matrices, namely :

(i) the diagonal,

$$M_R = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R, \quad (5)$$

and (ii) the off-diagonal

$$M_R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & c & 0 \\ 1 & 0 & 0 \end{pmatrix} m_R. \quad (6)$$

Moreover, in the diagonal model we will choose $b \simeq \epsilon^4$, $a \simeq \epsilon^5$, and in the off-diagonal model $c \simeq \epsilon^2$, as explained below. We also examine other cases of rank-3 matrices in the Appendix.

In the two cases (5) and (6), application of the seesaw formula gives the phenomenological viable form [12, 13]

$$M_\nu \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} m_\nu, \quad (7)$$

where, from now on, we denote

$$\epsilon = \epsilon_u \simeq 0.05 \quad (8)$$

This neutrino mass matrix corresponds to maximal mixing and a normal hierarchy with the overall scale $m_\nu \simeq 0.05$ eV, fixed by the atmospheric and accelerator neutrino oscillations, but with two different mass scales m_R .

The matrix (7) gives only a qualitative account for the experimental situation of neutrino masses and mixing, since it yields the square mass differences, $m_3^2 - m_2^2 \sim (0.1 \text{ eV})^2$, $m_2^2 - m_1^2 \sim 0$ and, after diagonalization of M_ν and M_e , a large lepton mixing.

It must be emphasized that our purpose in this paper is only an order-of-magnitude analysis, namely to examine the consistency between a neutrino spectrum and a lepton mixing matrix with approximately maximal mixing, and the amount of needed leptogenesis to explain the baryon asymmetry of the universe.

III. LEPTOGENESIS

Since we are interested in an order-of-magnitude calculation, we consider leptogenesis formulas in the single-flavor approximation [4]. The calculation must be done in the basis where the right-handed mass matrix is diagonal (with eigenvalues M_1, M_2, M_3). The baryon asymmetry, baryon to entropy fraction, is given by

$$Y_B \simeq \frac{1}{2} Y_L \quad (9)$$

and the lepton asymmetry by

$$Y_L \simeq 0.3 \frac{\epsilon_1}{g_*} \left(\frac{0.55 \cdot 10^{-3} \text{ eV}}{\tilde{m}_1} \right)^{1.16} \quad (10)$$

in the strong washout regime, and

$$Y_L \simeq 0.3 \frac{\epsilon_1}{g_*} \left(\frac{\tilde{m}_1}{3.3 \cdot 10^{-3} \text{ eV}} \right) \quad (11)$$

in the opposite weak washout regime. The parameter g_* is the number of light degrees of freedom, of the order $g_* \simeq 100$ in the standard case. Strong washout is realized for $\tilde{m}_1 \gg 3 \cdot 10^{-3}$, where $\tilde{m}_1 = (M_D^\dagger M_D)_{11}/M_1$.

Notice that Y_B is smaller than the baryon to photon ratio η by roughly a factor 7. The experimental value of the baryon asymmetry is (see [14]),

$$(Y_B)_{exp} \simeq 9 \cdot 10^{-11} \quad (12)$$

The CP-violating asymmetry ϵ_1 , related to the decay of the lightest right-handed neutrino is given here below case by case. We now consider the two different textures for the right-handed neutrino mass matrices proposed above.

IV. THE DIAGONAL MODEL

In the diagonal model (i), application of the seesaw formula gives the effective neutrino mass matrix

$$M_\nu \simeq \begin{pmatrix} -\epsilon^6/b + \epsilon^8 & i\epsilon^5/b + \epsilon^6 & i\epsilon^5/b + \epsilon^4 \\ * & -\epsilon^6/a + \epsilon^4/b + \epsilon^4 & i\epsilon^7/a + \epsilon^4/b + \epsilon^2 \\ * & * & \epsilon^8/a + \epsilon^4/b + 1 \end{pmatrix} \frac{m_t^2}{m_R}. \quad (13)$$

A structure similar to (7) is achieved for $b \sim \epsilon^4$ and $a \sim \epsilon^5$.

Since we hopefully expect

$$\frac{m_t^2}{m_R} \simeq 0.05 \text{ eV} \quad (14)$$

to describe the neutrino spectrum and lepton mixing, from (7) we get

$$m_R \sim 10^{15} \text{ GeV}, \quad (15)$$

near a unification scale. In this case

$$\epsilon_1 \simeq \frac{3}{16\pi v^2} \left(\frac{\text{Im}(M_D^\dagger M_D)_{12}^2}{(M_D^\dagger M_D)_{11}} \frac{M_1}{M_2} + \frac{\text{Im}(M_D^\dagger M_D)_{13}^2}{(M_D^\dagger M_D)_{11}} \frac{M_1}{M_3} \right), \quad (16)$$

where v is the v.e.v. of the Higgs field, and we get

$$\epsilon_1 \simeq 2 \cdot 10^{-9} \quad (17)$$

We have also

$$\tilde{m}_1 \simeq \epsilon m_\nu, \quad (18)$$

that lies in the weak washout regime, so that

$$Y_B \simeq 2 \cdot 10^{-12} \quad (19)$$

Note that $a \sim \epsilon^4$ is also possible. In such a case there is degeneracy in the lightest right-handed neutrinos and the lepton asymmetry is enhanced [15]. Note also that although we do not consider flavor effects, they can in principle preserve the asymmetry related to the decay of the second neutrino [16]; here $\epsilon_2 \sim 10^{-13}$.

V. THE OFF-DIAGONAL MODEL

In the off-diagonal model (ii), the seesaw formula gives

$$M_\nu \simeq \begin{pmatrix} -\epsilon^6/c & i\epsilon^7 + i\epsilon^5/c & \epsilon^8 + i\epsilon^5/c \\ * & i\epsilon^5 + \epsilon^4/c + i\epsilon^5 & \epsilon^6 + \epsilon^4/c + i\epsilon^3 \\ * & * & \epsilon^4 + \epsilon^4/c + \epsilon^4 \end{pmatrix} \frac{m_t^2}{m_R}. \quad (20)$$

A structure similar to (7) is now achieved for $\epsilon^4 \lesssim c \lesssim \epsilon^2$. For $c \simeq \epsilon^4$ one has $m_\nu \simeq m_t^2/m_R$, while in the more interesting case $c \simeq \epsilon^2$ one has

$$m_\nu \simeq \epsilon^2 \frac{m_t^2}{m_R} \simeq 0.05 \text{ eV}, \quad (21)$$

and hence, from $\epsilon \simeq 0.05 \text{ eV}$, the intermediate scale

$$m_R \sim 10^{12} \text{ GeV} \quad (22)$$

In this last case

$$\epsilon_1 \simeq \frac{3}{16\pi v^2} \left(\frac{\text{Im}(M_D^\dagger M_D)_{12}^2}{(M_D^\dagger M_D)_{22}} \frac{M_2}{M_1} + \frac{\text{Im}(M_D^\dagger M_D)_{23}^2}{(M_D^\dagger M_D)_{22}} \frac{M_2}{M_3} \right). \quad (23)$$

After a right-handed rotation in the 1-3 sector, we get for $c \simeq \epsilon^2$,

$$\epsilon_1 \simeq 5 \cdot 10^{-11} \quad (24)$$

With smaller values of the parameter c we obtain a larger scale m_R and a smaller amount of the CP asymmetry.

Notice one point. The simple ansatz (6) implies two degenerate very heavy Majorana neutrinos $M_1 = M_3 = m_R$ and a lighter one $M_2 \simeq \epsilon^2 m_R$. This is a quite different situation as the one considered in [6, 7], that needed a quasi-degeneracy of the lightest heavy neutrinos decaying out of equilibrium. Of course, the proposal (6) can be easily modified to have three heavy Majorana neutrinos of quite different masses.

VI. RELAXING QUARK-LEPTON SYMMETRY PRESERVING HIERARCHY OF MATRIX ELEMENTS

From the results of the preceding Sections, we realize that keeping strict quark-lepton symmetry (4) with M_u given by (2), both the diagonal and the off-diagonal models for the heavy Majorana right-handed neutrinos provide a too small baryon asymmetry.

Keeping the two models for the heavy Majorana right-handed neutrinos, we will now try to modify the quark-lepton symmetry relation (4), while preserving for the M_D matrix elements the same order of magnitude in powers of ϵ , i.e. we relax the quark-lepton symmetry relation while keeping the same hierarchy. Instead of $M_D = M_u$ with M_u given by (2), we propose then a Dirac neutrino mass matrix of the form

$$M_D \simeq \begin{pmatrix} 0 & O(\epsilon^3) & O(\epsilon^4) \\ O(\epsilon^3) & O(\epsilon^2) & O(\epsilon^2) \\ O(\epsilon^4) & O(\epsilon^2) & O(1) \end{pmatrix} m_t, \quad (25)$$

The interest of such an ansatz is that possible symmetries could link this matrix to M_u .

To have a guide about which scheme of this kind may give the right baryon asymmetry, we perform the following exercise. We modify the Dirac mass matrix M_D given by $M_D = M_u$ by putting i factors in several matrix elements. We make the trial

$$M_D \simeq \begin{pmatrix} 0 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} m_t \quad (26)$$

i.e. we drop the i factors in (2). Of course, this matrix is real and cannot lead to a CP asymmetry. However, this form can give us a hint of the possible interesting cases. With this last form of M_D we begin by computing the *real* quantities

$$\eta_1 \simeq \frac{3}{16\pi v^2} \left(\frac{(M_D^\dagger M_D)_{12}^2}{(M_D^\dagger M_D)_{11}} \frac{M_1}{M_2} + \frac{(M_D^\dagger M_D)_{13}^2}{(M_D^\dagger M_D)_{11}} \frac{M_1}{M_3} \right), \quad (27)$$

for the diagonal case (i), and

$$\eta_1 \simeq \frac{3}{16\pi v^2} \left(\frac{(M_D^\dagger M_D)_{12}^2}{(M_D^\dagger M_D)_{22}} \frac{M_2}{M_1} + \frac{(M_D^\dagger M_D)_{23}^2}{(M_D^\dagger M_D)_{22}} \frac{M_2}{M_3} \right). \quad (28)$$

for the off-diagonal case (ii).

For the diagonal model (i) we find $\eta_1 \simeq 10^{-8}$, while in the off-diagonal model (ii) this increases enormously, to $\eta_1 \simeq 10^{-4}$. This suggests that potentially the off-diagonal model is able to provide a sufficient amount of asymmetry, looking for appropriate complex models of M_D .

Following this guide-line, we do consider a model of the Dirac mass matrix with i factors in several matrix elements. Of course, we first have to check that the effective neutrino mass matrix is in agreement with the phenomenologically successful neutrino

mass matrix (7), and then calculate the baryon asymmetry. We find that adding i factors in positions 2-2, 2-3 and 3-2 this is viable. Therefore, our ansatz for the Dirac neutrino mass matrix is

$$M_D \simeq \begin{pmatrix} 0 & i\epsilon^3 & \epsilon^4 \\ i\epsilon^3 & i\epsilon^2 & i\epsilon^2 \\ \epsilon^4 & i\epsilon^2 & 1 \end{pmatrix} m_t \quad (29)$$

We turn now to the schemes (i) and (ii) for the heavy right-handed neutrino masses. Let us consider first the diagonal model (i). We find the neutrino mass matrix

$$M_\nu \simeq \begin{pmatrix} -\epsilon^6/b + \epsilon^8 & -\epsilon^5/b + i\epsilon^6 & -\epsilon^5/b + \epsilon^4 \\ * & -\epsilon^6/a - \epsilon^4/b - \epsilon^4 & i\epsilon^7/a - \epsilon^4/b + i\epsilon^2 \\ * & * & \epsilon^8/a - \epsilon^4/b + 1 \end{pmatrix} \frac{m_t^2}{m_R}. \quad (30)$$

Notice that using the values $b \sim \epsilon^4$ and $a \sim \epsilon^5$ proposed in Section IV to get a structure similar to (7), we have lost this structure adopting now the new form for M_D (27), since the (3,3) entry in (28) is small, of the order ϵ^3 . Therefore, maximal mixing can only be achieved in this case with some fine tuning. Moreover, one finds in this case a too small value of CP violation and baryon asymmetry, very close to (17) and (19).

Let us turn now to the more interesting off-diagonal model (ii). We find the neutrino mass matrix

$$M_\nu \simeq \begin{pmatrix} -\epsilon^6/c & i\epsilon^7 - \epsilon^5/c & \epsilon^8 - \epsilon^5/c \\ * & -\epsilon^5 - \epsilon^4/c - \epsilon^5 & i\epsilon^6 - \epsilon^4/c + i\epsilon^3 \\ * & * & \epsilon^4 - \epsilon^4/c + \epsilon^4 \end{pmatrix} \frac{m_t^2}{m_R}. \quad (31)$$

With $c \sim \epsilon^2$, as proposed in Section V, one gets at leading order in ϵ exactly the desired form (7) with $m_\nu \sim \epsilon^2(m_t^2/m_R)$, that suggests the intermediate scale (22). Moreover, one finds a larger amount of CP violation

$$\epsilon_1 \simeq 4 \cdot 10^{-7} \quad (32)$$

In this last case we have

$$\tilde{m}_2 = (M_D^\dagger M_D)_{22}/M_2 \simeq m_\nu, \quad (33)$$

a value that lies in the strong washout regime, so that

$$Y_B \sim 5 \cdot 10^{-12} \quad (34)$$

that is somewhat short of the order of magnitude (12). Remember that in the off-diagonal case the lightest right-handed neutrino corresponds to the second one in flavor.

However, we stress here that for

$$c \sim \epsilon^{3/2} \quad (35)$$

which is admissible, agreement with leptogenesis is improved, since now we have $m_\nu \sim \epsilon^{5/2} (m_t^2/m_R) \sim 0.05$ and we get the scale

$$m_R \sim 3 \cdot 10^{11} \text{ GeV}, \quad (36)$$

the CP asymmetry

$$\epsilon_1 \simeq 2 \cdot 10^{-6} \quad (37)$$

and therefore

$$Y_B \sim 1 \cdot 10^{-10} \quad (38)$$

that is of the right order of magnitude (12).

We have diagonalized the neutrino mass matrix in this latter case (31), and have checked that, to a good approximation, it gives the spectrum and mixing that follow from the simple ansatz (7).

To summarize, in the present context the off-diagonal model at the intermediate scale is preferred by leptogenesis with respect to the diagonal model at the unification scale.

VII. CONCLUSION

The present work relies on the phenomenological approach of textures. It is relevant to recall here that, for example within SO(10), it is possible to obtain any relation between quark and lepton mass matrices by using general Higgs representations, i.e. several **10s** and **126s**. However, we stress that the preservation of the hierarchical structure of mass matrices seems to point towards horizontal symmetries. Then it is also possible to generate relations between mass matrices [17].

In conclusion, we propose to relax the simple quark-lepton symmetry for the Dirac neutrino mass matrix $M_D = M_u$ and consider a kind of generalized quark-lepton symmetry that keeps the hierarchy of the matrix elements in terms of powers of ϵ . We could call this phenomenological approach *hierarchical* quark-lepton symmetry. In particular, we

have considered in detail a case in which the moduli of the matrix elements of M_D are equal to those of M_u .

Out of the several models, we have found one scheme consistent with the neutrino mass spectrum, lepton mixing and leptogenesis: the off-diagonal matrix (6) with one lightest right-handed neutrino corresponding to the second flavor, together with a Dirac mass matrix (29) with imaginary second row and column. The diagonal form is known to be viable only for degenerate lightest masses of the Majorana heavy neutrinos.

APPENDIX

We consider here two other interesting possibilities of rank-3 matrices for the heavy Majorana neutrinos :

$$M_R = \begin{pmatrix} d & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_R, \quad (\text{A.1})$$

$$M_R = \begin{pmatrix} 0 & e & 0 \\ e & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R. \quad (\text{A.2})$$

For these two models, the effective neutrino mass matrix is not in agreement with the structure (7). However, considering the real case, since the structure in sector 2-3 of the effective matrix is achieved and the smallness of the first row and column could be due to running effects [18], we perform again the calculation. In model (A.1) we get $d \sim \epsilon^4$ and $m_R \sim 10^{13}$ GeV and, moreover, $Y_L \sim 10^{-13}$. In model (A.2) we obtain $e \sim \epsilon^5$ and $m_R \sim 10^{16}$ GeV and the lepton asymmetry is enhanced due to degeneracy in the two lightest right-handed neutrinos [15].

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